JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 21, No. 6, November–December 1998

Output Feedback Variable Structure Adaptive Control of an Aeroelastic System

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The question of control of a class of aeroelastic systems with structural nonlinearities based on the variable structure model reference adaptive control theory is treated. It is assumed that the system parameters are unknown and unmodeled nonlinear functions are present in the model. Control laws for the trajectory control of pitch angle and plunge displacement are derived. For the synthesis of controllers, only measured output variable (pitch angle or plunge displacement) is used for feedback. The feedback loop includes two relays, whose outputs are functions of certain auxiliary error signals. Modulation functions of the relays are generated on-line using bounds on uncertain functions and signals derived from the input and output of the system. Digital simulation results are presented that show that in the closed-loop system with only output feedback, pitch angle, and plunge displacement are smoothly regulated to zero in spite of the uncertainty and unmodeled functions in the system.

Nomenclature

$A, \boldsymbol{b}_0, A_c, \boldsymbol{b}_c, \bar{\boldsymbol{b}}_0, H, F, \boldsymbol{g}$	= system matrices
a	= nondimensionalized distance from the midchord to the elastic axis
b	= semichord of the wing
c_h	= structural damping coefficient in plunge due to viscous damping
$c_{l_\alpha}, c_{m_\alpha}, c_{l_\beta}, c_{m_\beta}$	= lift, moment, and control coefficients
c_α	= structural damping coefficient in pitch due to viscous damping
e, e_0, e_i', y_a	= error signals
f_0, f_1	= modulation functions
$f(\alpha), f_c(\alpha)$	= nonlinear functions
h	= plunge displacement
I_α	= mass moment of inertia of the wing about the elastic axis
k_h	= structural spring constant in plunge
k_α	= structural spring constant in pitch
$k_\alpha(\alpha)$	= nonlinear stiffness
m	= mass
r	= input
U	= freestream velocity
W_m, L_1, G_p, G_d	= transfer functions
$\boldsymbol{x}, \boldsymbol{X}$	= state vectors
x_α	= nondimensionalized distance measured from the elastic axis to the center of mass
Z_p, R_p, Z_m, R_m	= polynomials in Laplace variable s
α	= pitch angle
β	= flap deflection
$\boldsymbol{\theta}, c_0, k, \boldsymbol{\theta}_{\omega_1}, \boldsymbol{\theta}_{\omega_2}, \theta_y, \theta_{ij}, \theta_{j\text{nom}}$	= parameters
ρ	= density of air
$\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3, \chi_i, \boldsymbol{\xi}_i$	= filtered signals

Introduction

AEROELASTIC systems exhibit a variety of phenomena including instability, limit cycle oscillations, and even chaotic vibrations.[1–3] Flutter is an oscillatory aeroelastic instability caused by unsteady aerodynamic loads.[1,2] Active control of aeroelastic systems is a problem of considerable interest. For flutter suppression, feedback control laws have been developed by researchers.[4–10] Piezoelectric actuation has been considered for flutter control in Refs. 10 and 11. In these studies, linear control theory has been used for the design of controllers. Based on a linear deterministic autoregressive moving average aeroservoelastic model, a digital adaptive controller for active flutter suppression has been derived in Ref. 12. Because the linear design is often not adequate, researchers have focused their attention on nonlinear aeroelastic models and developed control systems.[13–15] Although nonlinearities arising from control saturation, free play, hysteresis, structural stiffness, and stability derivatives are encountered in aeroelastic systems, nonlinear structural stiffness may play a dominant role in causing the onset of flutter.

Recently, experimental results have been obtained using an aeroelastic apparatus that permits a prescribed nonlinear structural stiffness.[15] In a series of interesting papers, Ko et al. have considered control of this aeroelastic system based on feedback linearization theory[16] and adaptive control technique.[17] The inverse controller design requires complete knowledge of the system parameters, and adaptive control law design requires knowledge of the structure of the nonlinear functions.[16,17] Furthermore, for the synthesis of controllers in Refs. 16 and 17, it is assumed that all of the state variables are available for feedback. Certainly, it is desirable to relax these stringent requirements for the design of aeroelastic control systems.

The contribution of this paper lies in the development of new control systems for the control of a nonlinear aeroelastic system based on variable structure model reference adaptive control (VSMRAC) theory.[18–20] It is assumed that the system has uncertain parameters, and moreover, the structure of the nonlinear functions in the model is unknown. Control systems for the trajectory control of plunge and pitch motion, using a trailing-edge flap, are designed. For the synthesis of these controllers, only the output variable (pitch angle or plunge displacement) for feedback is used. In the closed-loop system, the output variable (pitch angle or plunge displacement) asymptotically tracks the reference trajectory and converges to zero. It is noted that, unlike the customary adaptive schemes,[21] parameter divergence cannot occur in the closed-loop system because the control laws derived here do not use integral type adaptation laws.

It is pointed out that a VSMRAC system for the wing-rock control of a second-order model of slender delta wings has been recently designed.[22] However, the fourth-order aeroelastic system differs from the delta wing model in an important way as the aeroelastic system has zero dynamics of dimension two, but the delta wing model has no zero dynamics. It is well known in a qualitative sense that output trajectory control of systems that have exponentially stable zero dynamics is possible with internal stability in the closed-loop

Received Jan. 15, 1998; revision received May 4, 1998; accepted for publication May 5, 1998.

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system. However, for a successful application of the VSMRAC theory to aeroelastic systems with unknown dynamics, it is necessary to examine the transient behavior of the zero dynamics. Zero dynamics have significant effect on the convergence property of the state vector to its equilibrium value and the control input.

Aeroelastic Model

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are provided in Ref. 16 as

$$\begin{bmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{1}$$

where h is the plunge displacement and α is the pitch angle. In Eq. (1), M and L are the aerodynamic lift and moment. It is assumed that the quasisteady aerodynamic force and moment are of the form

$$\begin{aligned} L &= \rho U^2 bc_{l_\alpha}\left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right)b(\dot{\alpha}/U)\right] + \rho U^2 bc_{l_\beta}\beta \\ M &= \rho U^2 b^2 c_{m_\alpha}\left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right)b(\dot{\alpha}/U)\right] + \rho U^2 b^2 c_{m_\beta}\beta \end{aligned} \tag{2}$$

The nonlinear stiffness $k_\alpha(\alpha)$ is such that

$$\alpha k_\alpha(\alpha) = \alpha k_{\alpha 0} - k_{n_\alpha}(\alpha)$$

where $k_{\alpha 0}$ is a constant and $k_{n_\alpha}(\alpha)$ is the nonlinear part of αk_α. It is pointed out that although a rudimentary form of Eq. (1) is considered here, the VSMRAC theory is applicable to models with complex time-dependent nonlinearities and disturbance inputs. We shall be interested in the trajectories of Eq. (1) in a bounded region $\Omega \subset R^4$ surrounding the origin.

Consider a reference trajectory y_m that represents either a prescribed pitch angle trajectory α_m for pitch angle control or a plunge displacement trajectory h_m for plunge motion control. Appropriate reference trajectory is generated by a second-order command generator. We are interested in deriving control systems so that α tracks α_m or h tracks h_m asymptotically, and in the closed-loop system the state vector $(h, \alpha, \dot{h}, \dot{\alpha})^T$ converges to zero as $t \to \infty$. Here T denotes transposition.

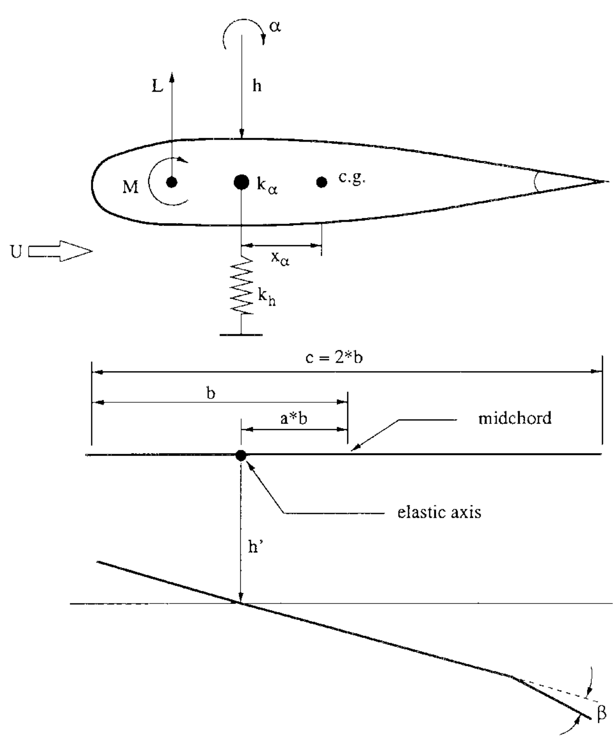


Fig. 1 Aeroelastic model.

VSMRAC Law

Input–Output Representations

The system described by Eq. (1) may be written as

$$D(s)\begin{pmatrix} h \\ \alpha \end{pmatrix} = \begin{pmatrix} -c_{l_\beta} \\ bc_{m_\beta} \end{pmatrix}\rho bU^2\beta + \begin{pmatrix} 0 \\ 1 \end{pmatrix}k_{n_\alpha}(\alpha) \tag{3}$$

where s is the Laplace variable or the differential operator and the elements of the matrix D are

$$\begin{aligned} d_{11} &= ms^2 + \left(c_h + \rho Ubc_{l_\alpha}\right)s + k_h \\ d_{12} &= mx_\alpha bs^2 + \rho Ub^2 c_{l_\alpha}(0.5 - a)s + \rho U^2 bc_{l_\alpha} \\ d_{21} &= mx_\alpha bs^2 - \rho Ub^2 c_{m_\alpha}s \\ d_{22} &= I_\alpha s^2 + \left[c_\alpha - \rho Ub^3 c_{m_\alpha}(0.5 - a)\right]s + k_{\alpha 0} - \rho U^2 b^2 c_{m_\alpha} \end{aligned} \tag{4}$$

Solving Eq. (3) for α and h gives

$$\begin{aligned} h &= -\left(d_{22}(s)c_{l_\beta} + d_{12}(s)bc_{m_\beta}\right)\Delta^{-1}(s)\rho bU^2\beta \\ &\quad - d_{12}(s)\Delta^{-1}(s)k_{n_\alpha}(\alpha) \\ \alpha &= \left(d_{21}(s)c_{l_\beta} + d_{11}(s)bc_{m_\beta}\right)\Delta^{-1}(s)\rho bU^2\beta \\ &\quad + d_{11}(s)\Delta^{-1}(s)k_{n_\alpha}(\alpha) \end{aligned} \tag{5}$$

where $\Delta(s) = d_{11}d_{22} - d_{12}d_{21}$. These input–output representations for h and α are useful for the design of controllers.

Pitch Angle Control

First, we shall consider control system design for the pitch angle control. The input β–output α representation obtained from Eq. (5) is written as

$$\alpha = G_{p_1}(s)\left[\beta + G_{d_1}(s)k_{n_\alpha}(\alpha)\right] \tag{6}$$

where the transfer functions G_{p_1} and G_{d_1} are given by

$$\begin{aligned} G_{p_1} &= \left\{m\left(x_\alpha c_{l_\beta} + c_{m_\beta}\right)s^2 + \left[c_{m_\beta}\left(c_h + \rho Ubc_{l_\alpha}\right) - \rho Ubc_{m_\alpha}c_{l_\beta}\right]s \right. \\ &\quad \left. + k_h c_{m_\beta}\right\}\rho b^2 U^2\left[m\left(I_\alpha - mb^2 x_\alpha^2\right)R_p\right]^{-1} \\ &\triangleq k_{p_1}\frac{Z_p(s)}{R_p(s)} \end{aligned} \tag{7}$$

$$G_{d_1} = \left[\frac{d_{11}(s)\Delta^{-1}(s)R_p(s)}{k_{p_1}Z_p(s)}\right]$$

Here $Z_p(s)$ and $R_p(s)$ are monic polynomials given by

$$Z_p(s) = s^2 + k_1 s + k_0 s$$

and

$$R_p(s) = (d_{11}(s)d_{22}(s) - d_{12}(s)d_{21}(s))\left[m\left(I_\alpha - mb^2 x_\alpha^2\right)\right]^{-1}$$

where

$$k_{p_1} = \frac{\left(x_\alpha c_{l_\beta} + c_{m_\beta}\right)\rho b^2 U^2}{\left(I_\alpha - mb^2 x_\alpha^2\right)}$$

$$k_1 = \frac{c_{m_\beta}\left(c_h + \rho Ubc_{l_\alpha}\right) - \rho Ubc_{m_\alpha}c_{l_\beta}}{k_d}$$

$$k_0 = \frac{k_h c_{m_\beta}}{k_d}$$

$$k_d = m\left(x_\alpha c_{l_\beta} + c_{m_\beta}\right)$$

Because k_{p_1} can be positive or negative, it will be convenient to define

$$\beta = \left(\operatorname{sgn} k_{p_1}\right) u \tag{8}$$

where u is a new input. Then Eq. (6) gives

$$\alpha = G_p(s)\left[u + G_d(s) k_{n_\alpha}(\alpha)\right]$$
$$\triangleq G_p(s)[u + f(\alpha)] \tag{9}$$

where $G_p(s) = k_p[Z_p(s)/R_p(s)]$, $k_p = |k_{p_1}| > 0$, $G_d(s) = \operatorname{sgn}(k_{p_1}) G_{d_1}(s)$, and $f(\alpha) = G_d(s) k_{n_\alpha}(\alpha)$.

Consider a reference model having input r and output α_m of the form

$$\alpha_m = W_m(s) r$$

where

$$W_m = k_m \frac{Z_m(s)}{R_m(s)} = \frac{k_m}{s^2 + p_{m_1} s + p_{m_0}} \tag{10}$$

with $Z_m = 1$. The polynomial $R_m(s)$ is assumed to be Hurwitz. A polynomial is said to be Hurwitz if its roots have a negative real part.

For the derivation of the VSMRAC, both the aeroelastic model and the reference model are assumed to satisfy the following conditions: 1) the monic polynomial $Z_p(s)$ of degree 2 is Hurwitz; 2) the sign of k_{p_1} is known; 3) the relative degree $n^* = \deg(R_p) - \deg(Z_p) = \deg(R_m) - \deg(Z_m) = 2$; and 4) $\operatorname{sgn}(k_p) = \operatorname{sgn}(k_m)$.

Condition 1 implies that $G_p(s)$ is a minimum-phase transfer function. This restriction on $Z_p(s)$ is essential for trajectory control even in nonadaptive control systems design.[16] The sign of k_{p_1} in condition 2 can be determined using a nominal model. Conditions 2 and 3 are easily satisfied by the choice of the reference model. The roots of $Z_p(s) = 0$ depend on the parameters of the system. The polynomial $Z_p(s)$ is Hurwitz if and only if k_0 and k_1 are positive. Because $c_{m_\beta} < 0$, polynomial $Z_p(s)$ is Hurwitz if and only if

$$\begin{aligned} k_d &= m\left(x_\alpha c_{l_\beta} + c_{m_\beta}\right) < 0 \\ c_{m_\beta}\left(c_h + \rho U b c_{l_\alpha}\right) &- \rho U b c_{m_\alpha} c_{l\beta} < 0 \end{aligned} \tag{11}$$

It is noted that $a \in (-1, 0)$. For the parameters given in Table 1, the first inequality in Eq. (11) is satisfied if $x_\alpha < -c_{m_\beta}/c_{l_\beta}$ which implies that $a > -0.542$. For these values of a, the second inequality holds for all values of U. We assume that the parameters of the model and the location of the elastic axis are such that these two conditions are satisfied, and therefore $G_p(s)$ is minimum phase. Note that k_{p_1} is negative because $k_d < 0$ and the moment of inertia $(I_\alpha - mb^2 x_\alpha^2)$ is positive. This implies that $\beta = -u$. Under the assumption that $G_p(s)$ has stable zeros, it follows from Eq. (7) that $G_d(s)$ is a stable transfer function, and it also follows from Eq. (7) that for a bounded function $\alpha(t)$, function $f(\alpha)$ is a bounded function.

Table 1 System parameters

Parameter	Value
b	0.135 m
k_h	2844.4 N/m
c_h	27.43 Ns/m
c_α	0.036 Ns
ρ	1.225 kg/m^3
c_{l_α}	6.28
c_{l_β}	3.358
c_{m_α}	$(0.5 + a)\, c_{l_\alpha}$
c_{m_β}	−0.635
m	12.387 kg
I_α	0.065 kgm^2
x_α	$[0.0873 - (b + ab)]/b$

Consider a controllable and observable representation of Eq. (9) as given by

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{b}(u + f(\alpha)), \qquad \alpha = H\boldsymbol{x} \tag{12}$$

where $G_p(s) = H(sI - A)^{-1}\boldsymbol{b}$. It is noted that the knowledge of matrices A, $\boldsymbol{b}$, and H is not essential for the design of the controller.

For the synthesis of the pitch angle controller, we introduce the following filters:

$$\dot{\boldsymbol{\omega}}_1 = F\boldsymbol{\omega}_1 + \boldsymbol{g}u, \qquad \dot{\boldsymbol{\omega}}_2 = F\boldsymbol{\omega}_2 + \boldsymbol{g}\alpha \tag{13}$$

where $\boldsymbol{\omega}_1, \boldsymbol{\omega}_2 \in R^3, \boldsymbol{g} \in R^3$,

$$F = \begin{bmatrix} -\lambda_2 & -\lambda_1 & -\lambda_0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \boldsymbol{g} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

and λ_i are the coefficients of

$$\Lambda(s) = \det(sI - F) = s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0 \tag{15}$$

Define $\boldsymbol{\omega} = [\boldsymbol{\omega}_1^T, \alpha, \boldsymbol{\omega}_2^T, r]^T \in R^8$.

For $f(\alpha) = 0$, there exists a unique constant vector $\boldsymbol{\theta}^* = [\boldsymbol{\theta}_{\boldsymbol{\omega}_1}^{*T}, \theta_y^*, \boldsymbol{\theta}_{\boldsymbol{\omega}_2}^{*T}, c_0^*]^T \in R^8$ with $\boldsymbol{\theta}_{\boldsymbol{\omega}_1}^* \in R^3$ and $\boldsymbol{\theta}_{\boldsymbol{\omega}_2}^* \in sR^3$ such that the transfer function of the closed-loop system with $u = \boldsymbol{\theta}^{*T}\boldsymbol{\omega}$ matches $W_m(s)$ exactly; i.e.,

$$\alpha = G_p(s)u = G_p(s)\boldsymbol{\theta}^{*T}\boldsymbol{\omega} = W_m(s)r \tag{16}$$

The parameter vector $\boldsymbol{\theta}^*$ satisfying Eq. (16) is obtained by solving[23]

$$\begin{aligned} &\boldsymbol{\theta}_{\boldsymbol{\omega}_1}^{*T}\gamma(s)R_p(s) + k_p\left(\boldsymbol{\theta}_{\boldsymbol{\omega}_2}^{*T}\gamma(s) + \theta_y^*\Lambda(s)\right)Z_p(s) \\ &\quad = \Lambda(s)[R_p(s) - Z_p(s)R_m(s)] \end{aligned} \tag{17}$$

$$c_0^* = k_m/k_p \triangleq (\kappa^*)^{-1}$$

where $\gamma(s) = [s^2, s, 1]^T$.

The adaptive controller, which is to be presented here, has many similarities with the controller designed in Ref. 22. Therefore, only the essential steps in the control design are given. For the derivation of the control law, one has to obtain a representation of the tracking error e_0 in suitable form. Defining $\boldsymbol{X} = (\boldsymbol{x}^T, \boldsymbol{\omega}_1^T, \boldsymbol{\omega}_2^T)^T \in R^{10}$, the system described by Eqs. (12) and (13) can be written as

$$\dot{\boldsymbol{X}} = A_0\boldsymbol{X} + \boldsymbol{b}_0 u + \bar{\boldsymbol{b}}_0 f, \qquad \alpha = H_c\boldsymbol{X} \tag{18}$$

where

$$A_0 = \begin{bmatrix} A & 0 & 0 \\ 0 & F & 0 \\ \boldsymbol{g}\boldsymbol{h}^T & 0 & F \end{bmatrix}, \qquad \boldsymbol{b}_0 = \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{g} \\ 0 \end{bmatrix}$$

$$\bar{\boldsymbol{b}}_0 = \begin{bmatrix} \boldsymbol{b} \\ 0 \\ 0 \end{bmatrix}, \qquad H_c = [H, 0]$$

Define

$$\begin{pmatrix} \boldsymbol{\omega}_1 \\ \alpha \\ \boldsymbol{\omega}_2 \end{pmatrix} = \begin{bmatrix} 0 & E & 0 \\ H & 0 & 0 \\ 0 & 0 & E \end{bmatrix} \boldsymbol{X} \triangleq N\boldsymbol{X}$$

Now using $\tilde{u} = u - u^* = u - \boldsymbol{\theta}^{*T}\boldsymbol{\omega}$ in Eq. (18) gives

$$\dot{\boldsymbol{X}} = A_c\boldsymbol{X} + \boldsymbol{b}_c\kappa^*\tilde{u} + \boldsymbol{b}_c r + \bar{\boldsymbol{b}}_0 f, \qquad \alpha = H_c\boldsymbol{X} \tag{19}$$

where $A_c = A_0 + \boldsymbol{b}_0[\boldsymbol{\theta}_{\boldsymbol{\omega}_1}^{*T}, \theta_y^*, \boldsymbol{\theta}_{\boldsymbol{\omega}_2}^{*T}]N$ and $\boldsymbol{b}_c = c_0^*\boldsymbol{b}_0$. For $u = u^*$ and $f = 0$, one has

$$W_m = H_c(sI - A_c)^{-1}\boldsymbol{b}_c \tag{20}$$

Therefore, the output of Eq. (19) takes the form

$$\alpha = W_m(s)r + \kappa^* W_m(s)\tilde{u} + f_c(z) \tag{21}$$

where

$$f_c(\alpha) = \bar{W}_m(s) f(\alpha), \qquad \bar{W}_m(s) = H_c(sI - A_c)^{-1}\bar{\boldsymbol{b}}_0 \tag{22}$$

Because $\bar{W}_m(s)$ is a stable operator, $f(\alpha)$ is bounded whenever α is a bounded function.

In view of Eq. (20), a nonminimal realization of the reference model is

$$\dot{\boldsymbol{X}}_m = A_c\boldsymbol{X}_m + \boldsymbol{b}_c r, \qquad \alpha_m = H_c\boldsymbol{X}_m \tag{23}$$

where $\boldsymbol{X}_m \in R^{10}$.

Let the state vector error be $\boldsymbol{e} = \boldsymbol{X} - \boldsymbol{X}_m$. Subtracting Eq. (23) from Eq. (19) gives the error equation

$$\dot{\boldsymbol{e}} = A_c\boldsymbol{e} + \boldsymbol{b}_c\kappa^*\tilde{u} + \bar{\boldsymbol{b}}_0 f, \qquad e_0 = H_c\boldsymbol{e} \tag{24}$$

where the pitch angle tracking error is $e_0 = \alpha - \alpha_m$. Using Eq. (24), the output tracking error can be written as

$$e_0 = \kappa^* W_m(s)\tilde{u} + f_c(\alpha) \tag{25}$$

Equation (25) is important for the derivation of the control law.

Table 2 VSMRAC algorithm

Variable	Definition
	Auxiliary errors
y_a	$= \kappa_{\text{nom}} W_m L_1 \left[u_0 - L_1^{-1} u_1\right]$
e_0	$= \alpha - \alpha_m$
e_0'	$= e_0 - y_a$
e_1'	$= (u_0)_{\text{eq}} - L_1^{-1}(u_1)$
	Modulation functions
f_0	$\geq \bar{\kappa}\left\lvert\chi_0 - \boldsymbol{\theta}_{\text{nom}}^T \xi_0\right\rvert + \sum_{j=1}^{8} \bar{\theta}_{0j}\left\lvert\xi_{0j}\right\rvert + \bar{g}_0 + \epsilon_0$
f_1	$\geq \sum_{j=1}^{8} \bar{\theta}_{1j}\left\lvert\xi_{1j}\right\rvert + \bar{g}_1 + \epsilon_1$
	Control law
u_i	$= f_i \operatorname{sgn}(e_i')$
i	$= 0, 1$
u	$= -u_1 + u_{\text{nom}} = \operatorname{sgn}\left(k_{p_1}\right)\beta$
u_{nom}	$= \boldsymbol{\theta}_{\text{nom}}^T \omega$

For the synthesis of the controller, it is essential to introduce a chain of auxiliary errors (e_i'). Because the relative degree n^* of the reference model is two, a polynomial $L_1(s)$ of degree $(n^* - 1)$ is chosen so that $W_m(s)L_1(s)$ is strictly positive real. We select $L_1(s)$ of the form

$$L_1(s) = (s + \lambda)/\lambda$$

where λ is a positive real number.

Now, we introduce the following set of filtered signals:

$$\chi_0 = L_1^{-1}\chi_1, \qquad \boldsymbol{\xi}_0 = L_1^{-1}\boldsymbol{\xi}_1$$

where $\chi_1 = u$, $\boldsymbol{\xi}_1 = \boldsymbol{\omega}$, and $\boldsymbol{\xi}_i = (\boldsymbol{\xi}_{i1}, \ldots, \boldsymbol{\xi}_{i8})$. These signals are used to generate a chain of auxiliary error signals e_i' $(i = 0, 1)$. Based on the results of Refs. 18–20, the complete algorithm for the aeroelastic control is given in Table 2.

In Table 2, $\boldsymbol{\theta}_{\text{nom}}$ and κ_{nom} are nominal values of the parameters $\boldsymbol{\theta}^*$ and κ^*, respectively, obtained from some nominal model of the plant. Also $\epsilon_0 > 0, \epsilon_1 > 0$, and the upper bounds $\bar{\theta}_{ij}$ $(i = 0, 1$ and $j = 1, \ldots, 8)$, $\bar{\kappa}$, and $\bar{g}_i$ $(i = 0, 1)$ for $\alpha \in \Omega_\alpha \subset \Omega$ are defined as

$$\bar{\theta}_{0j} > \rho\left\lvert\theta_j^* - \theta_{j\text{nom}}\right\rvert, \qquad \bar{\theta}_{1j} > \left\lvert\theta_j^* - \theta_{j\text{nom}}\right\rvert, \qquad \bar{\kappa} > |\rho - 1|$$

$$\bar{g}_0 > \sup_{t\geq 0}(k_{\text{nom}} W_m L_1)^{-1} f_c = \sup_{t\geq 0}(k_{\text{nom}} W_m L_1)^{-1} \bar{W}_m f \tag{26}$$

$$\bar{g}_1 > \sup_{t\geq 0} c_0^* W_m^{-1} f_c = \sup_{t\geq 0} c_0^* W_m^{-1} \bar{W}_m f$$

where θ_j^* denotes the jth element of $\boldsymbol{\theta}^*$ and $\rho = \kappa^*/\kappa_{\text{nom}}$. Note $(k_{\text{nom}} W_m L_1)^{-1}\bar{W}_m$ and $W_m^{-1}\bar{W}_m$ are strictly proper and proper stable transfer functions, respectively, and f is a bounded signal for α in a bounded set Ω_α. Also, $(u_0)_{\text{eq}}$ is the equivalent control, which can be formally obtained by setting $\dot{e}_0'(t) \equiv 0$ in the dynamical system governing the error e_0'. The signal $(u_0)_{\text{eq}}$ can be approximated from u_0 by means of a low-pass averaging filter.[24] Figure 2 shows the closed-loop system.

Now consider the VSMRAC law of Table 2. Then according to the results of Refs. 18–20 for any trajectory evolving in Ω, the closed-loop system has the following properties: 1) the errors e_i' $(i = 0, 1)$ all converge to zero in finite time and 2) the pitch angle tracking error e_0 converges exponentially to zero.

The signals $\boldsymbol{\omega}_i$, $i = 1, 2$, are used for computing the modulation functions. Although the control law of Table 2 is discontinuous, smooth approximations of the switching functions are used for the synthesis to avoid control chattering. For this purpose, one replaces $\operatorname{sgn}(\eta)$ by $\operatorname{sat}(\eta)$, where $\operatorname{sat}(\eta)$ is defined as $\operatorname{sat}(\eta) = \operatorname{sgn}(\eta)$ if $|(\eta)| > \delta$, and $\operatorname{sat}(\eta) = (\eta/\delta)$ if $|(\eta)| \leq \delta$. Here, δ is the

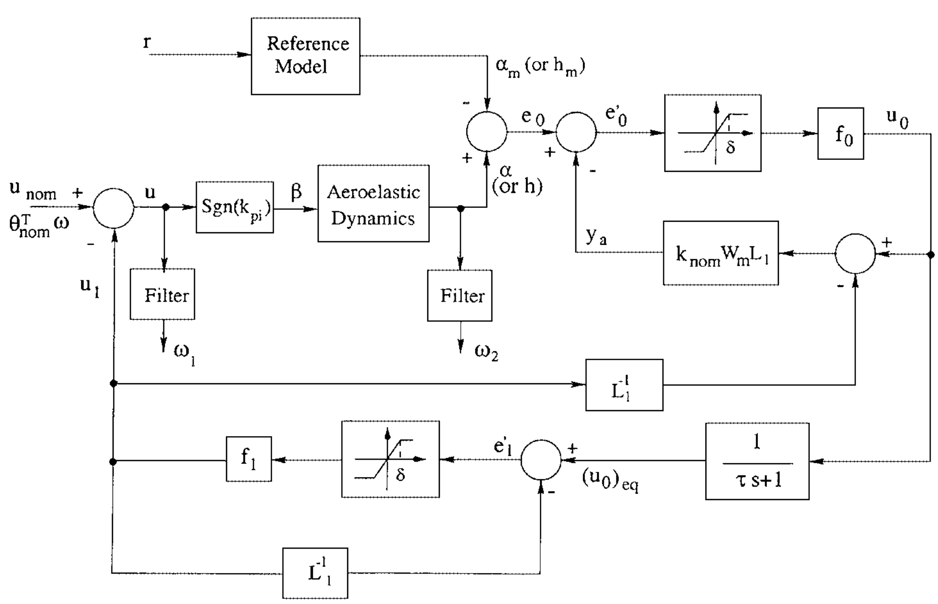


Fig. 2 Closed-loop system.

boundary-layer thickness. The lower bound functions of the modulation functions f_i are obtained on-line using the expressions in Table 2 and the available signals ξ_i and χ_0. Because the choice of the modulation functions satisfying the inequalities in Table 2 is only sufficient for stability in the closed-loop system, a practical way is to choose an overestimated value of bound $\bar{g}_i$ in the computation of the modulation functions, by observing simulated responses of an approximate aeroelastic model for a set of initial conditions. A simplified relay VSMRAC law is obtained when modulation functions are replaced by appropriate constants satisfying the conditions of Table 2.

It is noted that for the aeroelastic system, it is desired to regulate h and α to zero. Thus, one sets $r = 0$ in the reference model for generating a command input $\alpha_m(t)$ converging to zero. The VSMRAC law accomplishes regulation of pitch angle to zero. When the pitch angle is zero, the nonlinear function $k_{n_\alpha}(\alpha)$ vanishes. Setting $\alpha = 0$ and eliminating β in Eq. (5), the evolution of h is described by the zero dynamics of the system represented by a linear differential equation given by

$$(d_{21}(s)c_{l\beta} + d_{11}(s)bc_{m\beta})h = 0$$

which implies that

$$Z_p(s)h = 0$$

Because $Z_p(s)$ is assumed to be Hurwitz, it follows that $h(t) \to 0$ exponentially. Thus, in the closed-loop system the VSMRAC law accomplishes regulation of the state vector $(h, \alpha, \dot{h}, \dot{\alpha})^T$ to the origin in the state space.

Plunge Motion Control

For the plunge motion control, the output variable chosen is the plunge displacement $y = h$. In this case the input β–output h map is described by

$$\begin{aligned} h &= -\left(d_{22}(s)\Delta^{-1}(s)c_{l_\beta} + d_{12}(s)\Delta^{-1}(s)bc_{m_\beta}\right)\rho bU^2\beta \\ &\quad -d_{12}\Delta^{-1}(s)k_{n_\alpha}(\alpha) \\ &= G_{p_2}(s)\left[\beta + W_{d2}(s)k_{n_\alpha}(\alpha)\right] \end{aligned} \tag{27}$$

where

$$G_{p_2}(s) = -\left(d_{22}(s)c_{l_\beta} + d_{12}(s)bc_{m_\beta}\right)\Delta^{-1}(s)\rho U^2 b \triangleq k_{p_2}\frac{Z_{p_2}(s)}{R_p(s)}$$

and

$$G_{d_2}(s) = -\frac{R_p(s)d_{12}(s)\Delta^{-1}(s)k_{n_\alpha}(\alpha)}{k_{p_2}Z_{p_2}(s)} \tag{28}$$

The monic polynomial Z_{p_2} of second degree is easily computed from Eq. (27). The high-frequency gain k_{p_2} is given by

$$k_{p_2} = \frac{-\rho bU^2\left(I_\alpha c_{l_\beta} + b^2c_{m_\beta}mx_\alpha\right)}{mI_\alpha - m^2b^2x_\alpha^2}$$

The control input β is defined as

$$\beta = \mathrm{sgn}(k_{p_2})u$$

where u is a new input. For the derivation of the VSMRAC, it is assumed again that $G_{p_2}(s)$ is minimum phase. The derivation of the control law is done in a similar manner as described in the preceding section and, therefore, is not repeated here. The control algorithm is obtained from Table 2 by replacing the pitch angle tracking error by $e_0 = h - h_m$, where h_m is the reference plunge displacement trajectory. In this case, modulation functions f_i depend on α. Overestimated constant modulations are used for the synthesis of the controller. Of course, the lower bounds of the modulation functions have different expressions, which are derived following the steps outlined in the preceding section.

Here $h_m(t)$ is a reference trajectory converging to zero generated by the reference model of Eq. (10) with $r = 0$. The VSMRAC accomplishes tracking of $h_m(t)$ and regulation of $h(t)$ to zero. In the closed-loop system, when $h(t) \equiv 0$, it is seen from Eq. (3) that the zero dynamics are described by

$$\begin{aligned} d_{12}(s)\alpha &= -\rho U^2bc_{l_\beta}\beta \\ d_{22}(s)\alpha &= \rho U^2b^2c_{m_\beta}\beta + k_{n_\alpha}(\alpha) \end{aligned} \tag{29}$$

Eliminating β, the nonlinear zero dynamics are given as

$$\left[bc_{m_\beta}d_{12}(s) + c_{l_\beta}d_{22}(s)\right]\alpha - c_{l_\beta}k_{n_\alpha}(\alpha) = 0 \tag{30}$$

Because, by assumption, $G_{p_2}(s)$ in Eq. (28) is minimum phase, in view of Eq. (27), the polynomial

$$n_p(s) \triangleq \left[bc_{m_\beta}d_{12}(s) + c_{l_\beta}d_{22}(s)\right]$$

is Hurwitz. Substituting the expressions for d_{12} and d_{22}, it easily follows that $n_p(s)$ is Hurwitz if and only if

$$\begin{aligned} &\frac{U^2b^2\rho\left(c_{l_\alpha}c_{m_\beta} - c_{m_\alpha}c_{l_\beta}\right) + k_{\alpha 0}c_{l_\beta}}{k_{d2}} > 0 \\ &\frac{c_\alpha c_{l_\beta} + b^3\rho U(0.5 - a)\left(c_{m_\beta}c_{l\alpha} - c_{m_\alpha}c_{l_\beta}\right)}{k_{d2}} > 0 \end{aligned} \tag{31}$$

where

$$k_{d2} = \left(I_\alpha c_{l_\beta} + b^2mx_\alpha c_{m_\beta}\right)$$

For the values of parameters in Table 1, k_{d2} is positive. It is easily verified that for $-1 < a \leq -0.5 + (c_{m_\beta}/c_{l_\beta}) = -0.6891$,

$$c_{l_\alpha}c_{m_\beta} - c_{m_\alpha}c_{l_\beta} \geq 0 \tag{32}$$

and both the inequalities in Eq. (31) hold for all values of U. But for $-0.6891 < a < 0$, these inequalities in Eq. (31) are satisfied only if the velocity U satisfies

$$U < \min\left\{\left(\mu^*k_{\alpha 0}\right)^{\frac{1}{2}}, c_\alpha\mu^*[b(0.5 - a)]^{-1}\right\} \tag{33}$$

where $\mu^* = c_{l_\beta}/[b^2\rho(c_{m_\alpha}c_{l_\beta} - c_{l_\alpha}c_{m_\beta})]$. The stable region in the a–U plane satisfying Eq. (33) is shown in Fig. 3. Note that for $-1 < a \leq -0.6891$, the zero dynamics are stable for any U, and in Fig. 3, U tends to infinity as a tends to -0.6891 from the right.

For minimum phase G_{p_2}, the equilibrium point $(\alpha, \dot{\alpha}) = 0$ of Eq. (30) is locally exponentially stable. This implies that as the VSMRAC accomplishes regulation of $h(t)$ to zero, the pitch angle trajectory, which remains in a small neighborhood of the origin, also tends to zero asymptotically. It is evident from the zero dynamics equation (30) that, in this case (unlike pitch angle control), one expects somewhat complex dynamic behavior of the pitch angle.

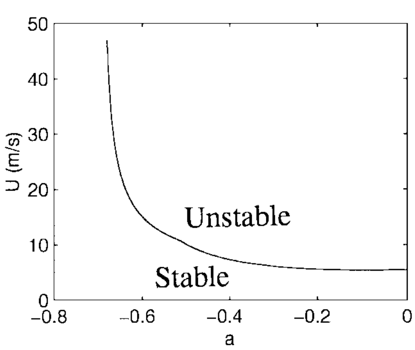


Fig. 3 Region in a–U plane for stable zero dynamics for plunge control.

Simulation Results

In this section, numerical results for the pitch angle and plunge motion control are presented. The closed-loop system equation (1) with the controller of Table 2 is simulated. The parameters of the system are given in Table 1. Simulation is performed for different values of a and U. The transfer function of the command generator is chosen as

$$W_m = \frac{\lambda^2}{(s+\lambda)^2}$$

to obtain exponentially decaying command trajectories. The initial conditions selected are $h(0) = 0.01$ m, $\alpha(0) = 5.73$ deg, and $(\dot{h}(0), \dot{\alpha}(0)) = 0$. The initial conditions of the command generator are set as $\alpha_m(0) = \alpha(0)$, $\dot{\alpha}_m(0) = 0$ for the pitch angle control and $h_m(0) = h(0)$, $\dot{h}_m(0) = 0$ for the plunge motion control. The value of modulation functions f_i, the boundary-layer thickness δ, and the parameter τ of the averaging filter are selected after several trials by observing the simulated responses. For simulation, it is assumed that $|\beta| \leq 20$ deg and β is set to its limiting values whenever it exceeded the prescribed limits. The maximum tracking error is denoted as e_{om}. Figure 4 shows the response of the open-loop system ($\beta = 0$, $a = -0.4$, and $U = 15$ m/s). For the uncontrolled system, α and h undergo persistent periodic oscillations.

Case A1

For case A1, the closed-loop system for $a = -0.4$ and $U = 15$ m/s is simulated. The controller parameters are selected as $\lambda = 0.4$, $f_0 = f_1 = 500$, $k_{\text{nom}} = 30$, $\tau = 0.001$, and $\delta = 0.5$. The responses are shown in Fig. 5. We observe smooth pitch angle tracking control in about 18–20 s. Because the $G_{p_1}(s)$ is minimum phase, plunge displacement also converges to zero. The maximum tracking error $(\alpha - \alpha_m)$ is 1.3 deg.

Case A2

Simulation is done for the model of case A1, but higher freestream velocity $U = 20$ m/s is assumed. Because responses are somewhat similar to those in Fig. 5, these results are not shown here. Inasmuch as the control effectiveness matrix is larger, small control input (less than 13 deg is required in this case. The maximum error e_{om} is less then 0.6 deg.

Case A3

Simulation is done for the model of case A2, but a different value of $a = -0.45$ is chosen. The selected response is shown in Fig. 6. Although, a larger tracking error compared to case A2 (about 3.3 deg) is observed, regulation of both the pitch angle and the plunge displacement to zero is accomplished.

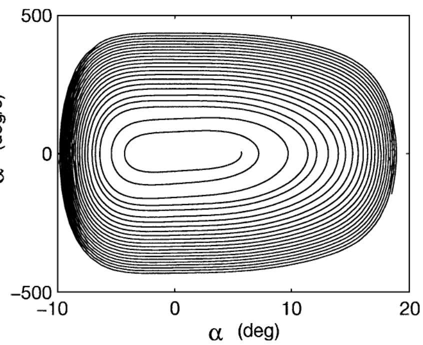


Fig. 4 Response of uncontrolled system.

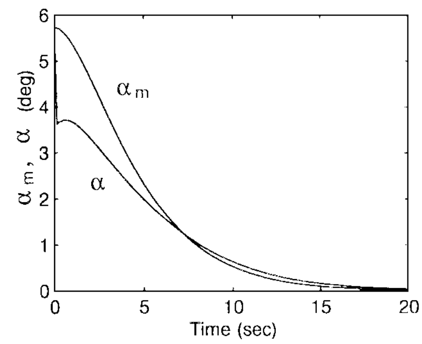
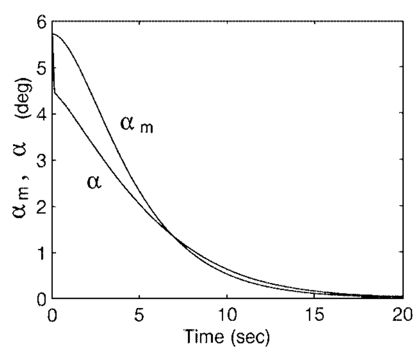
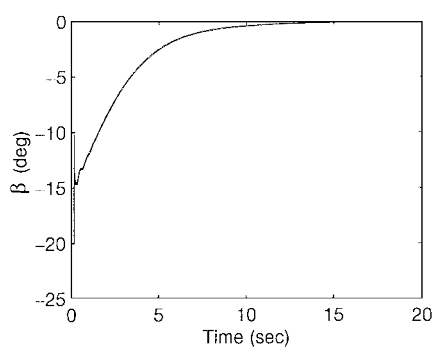


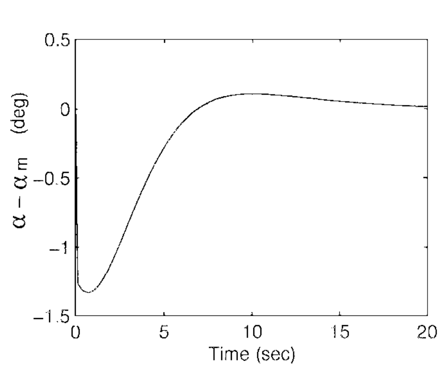
Fig. 6 Pitch angle control, $a = -0.45$ and $U = 20$ m/s.



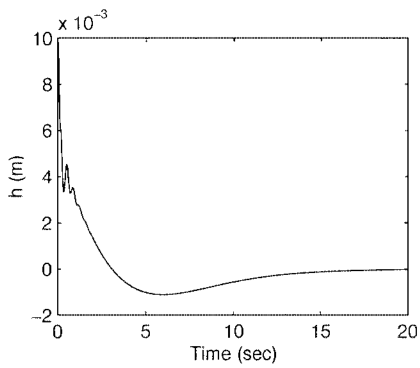
a) Pitch angle α and α_m



c) Control input β

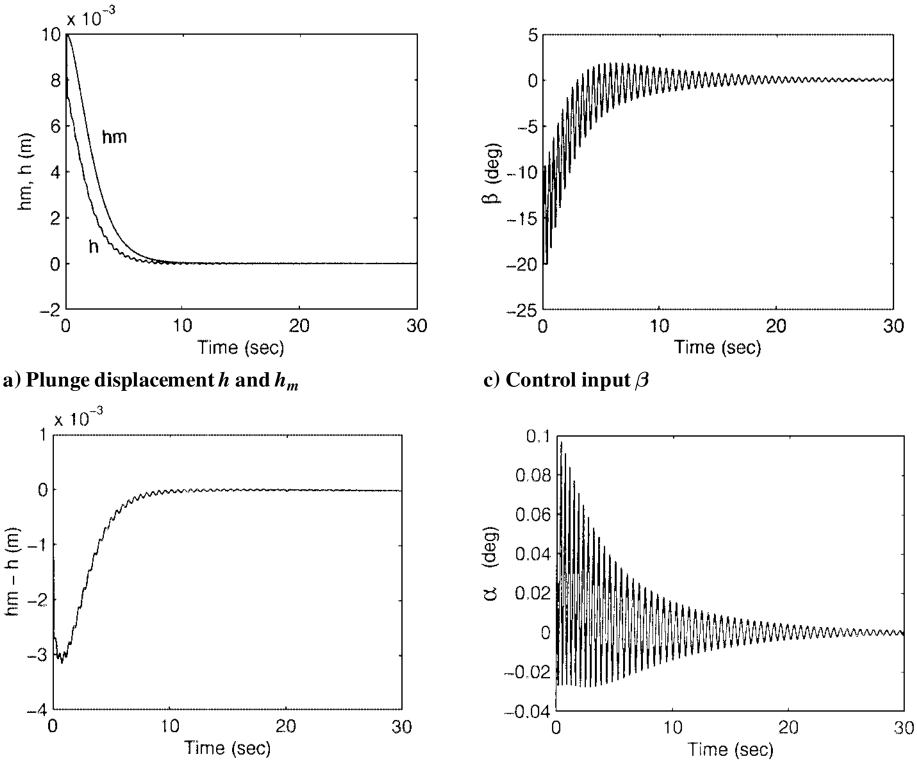


b) Tracking error $e_0 = \alpha - \alpha_m$



d) Plunge displacement h

Fig. 5 Pitch angle control, $a = -0.4$ and $U = 15$ m/s.



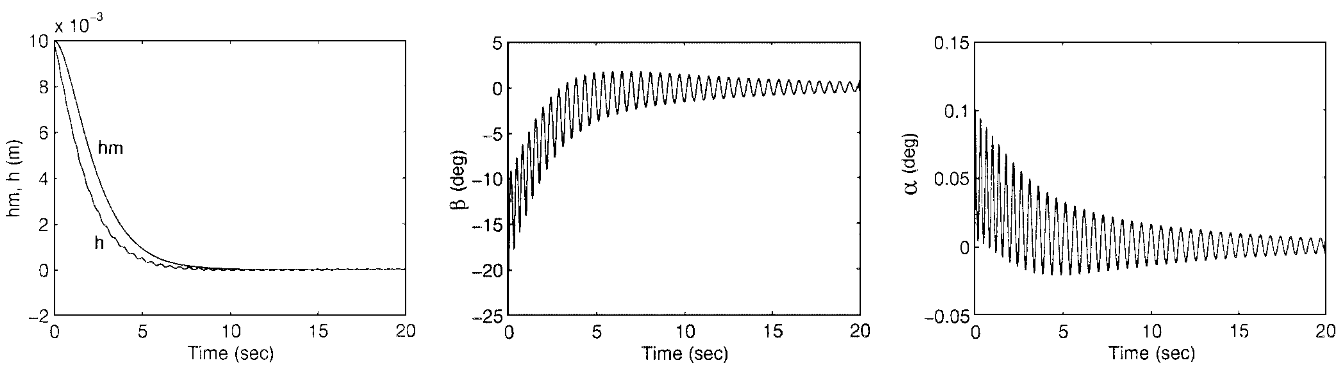
a) Plunge displacement h and h_m

b) Tracking error $e_0 = h - h_m$

c) Control input β

d) Pitch angle α

Fig. 7 Plunge motion control, $a = -0.68$ and $U = 15$ m/s.



a) Plunge displacement h and h_m

b) Control input β

c) Pitch angle α

Fig. 8 Plunge motion control, $a = -0.75$ and $U = 15$ m/s.

Case B1

The closed-loop system for plunge motion control with $a = -0.68$ and $U = 15$ m/s is simulated in which the controller given in Table 2 with $e_0 = h - h_m$ is synthesized. The controller parameters are chosen to be $\lambda = 0.8$, $k_{\text{nom}} = 6$, $f_0 = f_1 = 500$, $\tau = 0.001$, and $\delta = 0.5$. The model parameters are chosen as $a = -0.68$ and $U = 15$ m/s such that the zero dynamics are stable. For these values, the transfer function $G_{p_2}(s)$ is minimum phase. Responses are shown in Fig. 7. The plunge displacement smoothly follows the command trajectory and converges to zero. The response time for h is quicker because a faster command generator ($\lambda = 0.8$) has been used. Because the zero dynamics are nonlinear, oscillatory but convergent α response is observed in this case.

Case B2

Simulation is done using the model of case B1, but the speed U is increased to 20 m/s. A smaller control input (less than 18 deg) and tracking error $e_0 = (h - h_m)$ (less than 2.1×10^{-3} m) are observed. The state vector asymptotically converges to zero. Because the responses are somewhat similar to those of case B1, case B2 responses are not shown.

Case B3

The model of case B1, with $a = -0.75$ and $U = 15$ m/s, is simulated. For this value of a, the zero dynamics are stable. Selected responses are shown in Fig. 8. Again convergent responses for the pitch and plunge motion are observed. Compared to case B1, the tracking error is relatively small (less than 2.1×10^{-3} m).

Conclusions

Active control of an aeroelastic system is presented. Based on the variable structure adaptive model reference control theory, control systems for pitch angle and plunge displacement trajectory control are derived. Interestingly, for the synthesis of the control systems, only the tracking error is required, and the knowledge of system parameters and the nonlinear model structure is not assumed. This is an important feature because the nonlinear stiffness of model is difficult to model. In the closed-loop system, tracking error asymptotically tends to zero, and the state vector is regulated to the origin. Simulation results are presented that show that, in the closed-loop system, precise trajectory control of pitch angle and plunge displacement is accomplished. Furthermore, it is seen that regulation of the state variables to the origin is accomplished in spite of the unmodeled structural nonlinearities and parameter uncertainties in the aeroelastic model.

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